Estimating Demand Elasticities using the PCAIDS Method

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1 INTRODUCTION

Difficult economic conditions in Europe and worldwide, along with continuing pressure from competitive entry, electronic substitution, and other factors, continue to put pressure on Posts and mail volumes. Of key importance to the continued operation of the Postal market and the USO is pricing regulation that balances cost reflectivity, the need to finance the USO, and the need to meet competition dynamically. Some of the more interesting issues and challenges within this context are that some mail products might be facing negative impacts from e-commerce (e-substitution – e.g., small letters), while other products might be more complimentary with e-commerce (e-complementarity – e.g., online ordering of packages). The ambiguous impact of this and other facets of e-substitution illustrates the continuous need to estimate the drivers of mail demand and the sensitivity of mail demand to price changes.

Within the context of these challenges, one of the more important parameters to estimate for the postal economist is the price elasticity of demand (Robinson 2007). Naturally, as there is in fact a host of postal products, there is a corresponding host of price elasticity parameters that must be estimated. Product definition can vary by a large number of parameters, including: weight, format, class (speed), franking method, volume, and delivery receipt. While there has been a large body of research developing on postal price elasticity estimation, a challenge that still remains in price elasticity estimation is that often there are too many products to reliably estimate elasticities for all the existing products, given the data usually available. An alternative to directly estimating all the parameters involved is to use a model such as the Proportionality Calibrated Almost Ideal Demand System (PCAIDS) model (Epstein and Rubinfeld 2001).

This paper applies the proportionality calibrated almost ideal demand system (PCAIDS) method to estimate a full set of demand elasticity parameters for a set of postal products. The PCAIDS model is based on the same principles as the linear-approximate demand model (LA-AIDS) model, but requires less information to be estimated than the LA-AIDS model and estimates cross-price estimates based on an analytical formula, derived and endogenous to observed market shares, the own-price elasticity of one product in the market and an aggregate price elasticity of the industry.

The paper uses data from An Post's regulatory accounts and published sources.

The rest of this chapter is organized as follows: Section 2 contains a literature review of the models. Section 3 shows examples of results using the models, and Section 4 gives a summary and our conclusions from the results.

2 LITERATURE REVIEW

2.1 AIDS model

Demand estimation in postal pricing has a long history and can take many forms. When estimating various price relationships between variables in the postal sector, it is important to include the likely cross-price effects between different postal products in the sector. To obtain own and cross price elasticity parameter estimates is often useful to make use of demand theory, and thus estimate a system of demand equations. One of the most common methods is to use the so-called Almost Ideal Demand System (AIDS). The AIDS model, developed by Deaton and

Muellbauer (1980)[1] is a system of equations approach to demand estimation. The demand system approach is useful in the context of estimating a consistent set of own- and cross-price elasticities, since it ensures that the full set of parameters will be consistent with certain restrictions implied by the neoclassical model of consumer demand, such as linear homogeneity in prices and income.

The AIDS model is usually estimated using standard econometric methods.[2] It is common to simplify the model into a linear approximate (LA-AIDS) (Deaton and Muellbauer 1980, 1999) model or a quadratic model (QUAIDS) (Green and Alston (1991)).

The fundamental equations of the AIDS model are shown in Equation 1. The model allows for non-linearity between the demand shares and the various prices. However, the model is often estimated by linearizing the model. This is done by estimating an aggregate price index. This transformation is the basis for the estimation of the linear approximate almost ideal demand system (LA-AIDS).

Consumer expenditures on particular products are broken down into a system of equations—namely, a (log) total expenditure equation (ln*x*), and expenditure-share equations (s_i). The system of equations is then estimated simultaneously as an iterative seemingly unrelated regression (iSUR) system with constraints. The AIDS model can be written as follows:[3]

Equation 1:

$$\ln x = a + \sum_i \alpha_i \ln p_i + \tfrac{1}{2}\sum_j \sum_i \gamma_{ij} \ln p_i \ln p_j + b \prod_i p_i^{\beta_i}$$

$$s_i = \alpha_i + \sum_j \gamma_{ij} \ln p_i \ln p_j + \beta_i \ln\left(\frac{x}{P^*}\right)$$

In the equation above: ln is the natural log; the p_i are the prices of the postal products, (stamp, meter, bulk), the s_i is the ith product's share in revenue, x is total revenue, P^* is a general price index, and the lowercase Greek letters are the parameters to be estimated.

The system is made more tractable (reducing the number of free parameters to be estimated) by imposing restrictions within and across equations, including symmetry, homogeneity, and adding up (budget shares must sum to one). The restrictions imply:

Equation 2: $$\sum_j \gamma_{ij} = 0,\ \sum_i \beta_i = 0,\ \sum_i \gamma_{ij} = 0,\ \gamma_{ij} = \gamma_{ji},\ \sum_i \alpha_i = 1$$

From the above system of equations and parameter estimates, the own- and cross-price elasticity estimates can be derived. In these equations, ε_{ii} refers to the own-price elasticity for the various products and ε_{ij} refers to the cross-price elasticity between product *i* and product *j*.

[1] Deaton & Muellbauer (1980).

[2] See e.g., Green & Alston (1990).

[3] Ibid. at note 1.

Equation 3:

$$\varepsilon_{ii} = \frac{1}{\bar{s}_i}\left(\gamma_{ij} + s_i^b \beta_i\right) - 1$$

$$\varepsilon_{ij} = \frac{1}{\bar{s}_i}\left(\gamma_{ij} + s_j^b \beta_i\right)$$

While generally useful, LA-AIDS-type models have certain limitations. Generally, they are considered to have sound economic fundamentals, which are based in neoclassical consumer behaviour theory. However, they can often lead to statistically insignificant parameter estimates and unexpected signs on explanatory variables and the results may be sensitive to some of the approximations needed. The choice of a price index for the final term, P*, in Equation 1 is just one example. These problems often arise due to data constraints, where the number of observations is limited in size, especially relative to the number of parameters that must be estimated.

Problems with econometric estimation of the AIDS or LA-AIDS models can become severe, especially when there are a great number of products and product categories. The postal products array suffers precisely from this problem; that is, there are actually a great number of products and sub-products, with prices and demands by weight-step, speed-of-delivery/class, franking type, and format. If one were to try and implement or estimate an econometric model of demand that estimated the total number of parameters, data limitations and a lack of degrees of freedom would likely render the estimates insignificant.

An alternative option for estimation when cross-price elasticities and more refined and granular analysis of postal prices and product demand elasticities is needed is to use the proportionality calibrated almost ideal demand system, or PCAIDS model developed by Epstein and Rubinfeld (2001).[4] The PCAIDS model uses as inputs an aggregate demand elasticity estimate, and then estimates a variety of other parameters needed to characterize the complete demand system of products using restrictions on the system implied by demand theory, and as the name implies, a proportionality calibration method.

The PCAIDS model was introduced in 2001 by Epstein and Rubinfeld (2001) as an improvement over other methods of demand elasticity analysis in the context of measuring the effects of mergers for competition analysis with regards to both effects and remedies. The authors refined the original Almost Ideal Demand System (AIDS) (Deaton and Muellbauer 1980).[5] The AIDS-type models provide a framework for analysis of own-price elasticities and cross-price elasticities and their effects on market share of firms producing the same or substitute goods. The PCAIDS model is useful because it allows for estimations to be achieved when: 1) data is limited, 2) estimation is problematic and/or 3) time and cost limitations prohibit a more in-depth analysis.[6]

The PCAIDS model is a two-parameter model and thus the number of parameters to be estimated is greatly reduced vìs-a-vìs a full AIDS model or similar methods. The PCAIDS model requires estimates (as inputs) of the following parameters: own-price industry elasticity, own-price

[4] Epstein & Rubinfeld, (2001).

[5] Deaton & Muellbauer (1980).

[6] Ibid. at note 1.

elasticity of one product in the market, nesting matrix (optional), markets shares for all products (or products and brands) in the industry.

Epstein and Rubinfeld (2001) describe the value of their model in very basic terms:

> "It requires information only on market shares, the industry price elasticity, and the price elasticity for one brand in the market. The logic of PCAIDS is simple. The share lost as a result of a price increase is allocated to the other firms in the relevant market in proportion to their respective shares."[7]

By relying solely on data on market share and price elasticity estimates (available pre-merger in the antitrust application, or before a price change—in the case of postal regulatory applications), simulation models (and the PCAIDS model, specifically) are very useful for competition and regulatory authorities wishing to evaluate the possible effects of a proposed merger or a price change.[8] Other models may require the use of post-change regulatory data to evaluate the market effects of a merger or policy change, which can be impractical.[9]

The use of the PCAIDS model has taken hold among competition authorities. For example, the UK's Office of Fair Trading (OFT) uses the PCAIDS model, where appropriate, to evaluate mergers *ex ante* in differentiated product industries, and a conservative estimate is used for any assumptions made in terms of consumer benefit.[10] The OFT uses a set of assumptions and estimates in choosing which simulation model to use and has stated:

"Given the data we usually have available, the economic models we have used are, in our opinion, the best academically supported method for mechanically deriving estimates of the impact of a merger decision for the purpose of impact estimation after a potential Substantial Lessening of Competition (SLC) has been made by the Office."[11] However, outside of the application of the PCAIDS model to the most recent pricing application from An Post, the Irish USP to their regulator ComReg, we are not aware of similar regulatory applications.[12]

2.1.1 The PCAIDS model

The PCAIDS model is an extension of the typical AIDS model. The assumptions that were applied previously in the AIDS model are also applied in the PCAIDS framework. There are three additional assumptions that are imposed in a PCAIDS model. These additional constraints are proportionality, adding-up and homogeneity. The most important of these restrictions in terms of its impact on various elasticity estimates is the assumption of proportionality. Under proportionality, the PCAIDS model imposes the assumption that sales are diverted away from a product according to the relative market share of the suite of products in the defined market.

[7] Ibid.

[8] Ivaldi & Verboven (2005).

[9] Neven & Zenger (2008).

[10] Davies (2010).

[11] Jones & Stallibrass, (2007).

[12] See e.g., An Post (2012).

As the PCAIDS model is a variant of the standard AIDS model, it may be expressed as a system of equations as in Equation 4. It should be noted here that the PCAIDS model does not include any AIDS expenditure terms. Thus the share equations below refer to the share of the product whose elasticity is known and a share for all other products in the market. The market shares in this model can be the average of the revenue shares at the end of the sample period. In this case, there is no need for detailed monthly data, which might be used to obtain more degrees of freedom for econometric estimation. This is the main advantage of PCAIDS as it has much lower data requirements when compared with other demand models like an AIDS-type model or a very data intensive model like the random coefficients logit model.

Equation 4:

$$S_1 = a_1 + b_{11}\ln(p_1) + b_{12}\ln(p_2) + \ldots + b_{1,N-1}\ln(p_{N-1})$$

$$S_{N-1} = a_{N-1} + b_{N-1,1}\ln(p_1) + b_{N-1,2}\ln(p_2) + \ldots + b_{N-1,N-1}\ln(p_{N-1})$$

In Equation 4, $\ln(p_1)$ refers to the natural log of the price of Product 1. As part of the PCAIDS model, a price elasticity for this one product in the market must be estimated separately. In this equation, the coefficient on b_{11} must be converted into an elasticity using Equation 5 (below). By imposing the adding-up and homogeneity constraints, we are able to derive the various elasticities with reference to the estimated coefficients. These are shown in Equation 5. The coefficient b_{jj} refers to the coefficient on the price variable in the share equation that is estimated in the two-product LA-AIDS model.

Equation 5:

$$\varepsilon_{jk} = b_{jk}/s_j + s_k(1+\varepsilon)$$

$$\varepsilon_j = b_{jj} - 1 + s_j(1+\varepsilon)$$

The value in this approach is that once the own-price elasticity is estimated for any one product within the market, then this estimate can be used to estimate all other own-price elasticities in the market once the revenue shares are known for all products. These can be derived by applying the equations shown in Equation 6 where ϵ_1 is the price elasticity of one product in the market. ϵ refers to the price elasticity of the overall market.

Equation 6:

$$\varepsilon_{jk} = s_k(\varepsilon - \varepsilon_1)/(1 - s_1)$$

$$\varepsilon_j = [(1 - S_j)\varepsilon_1 + (s_{j-}\ {}_{S_1)\varepsilon}]/(1 - S_1)$$

A more detailed derivation of PCAIDS models than is presented in this report is given in Coloma (2006).[13]

2.1.2 Derivation of PCAIDS (with nests)

In many-product markets, one has significant amounts of *a priori* information regarding substitution patterns within a market. This could be informed by particular product attributes which makes substitution between more comparable products more likely. Using a standard PCAIDS approach,[14] the shares will be diverted proportionality to the other products in the market based on their observed market shares. However, this does not take into account the potential similarity between products in the market. Using a nesting structure, we can overcome this problem whereby more weight is given to products with similar characteristics.

In Epstein and Rubinfeld (2004), the example given is in relation to different brands of beer. A nesting structure is applied to the various light beers products in the overall market. This is based on the prior expectation that these light beers are closer substitutes to each other than non-light beers. A similar approach can be adapted to many other products markets where there are observable differences between the products within the market.

The only other parameter needed to estimate the PCAIDS model with nests are the nesting parameters themselves. The PCAIDS elasticities with nests are derived using the coefficients estimated in Equation 7 along with the formulas for deriving the standard PCAIDS model shown in Equation 5. The standard procedure to compute PCAIDS with nests is to invert Equation 5 which gives the coefficient b_{11}. Putting this coefficient into Equation 7 allows us to estimate the remaining b coefficients in the demand system. Using these b coefficients and Equation 5, allows us to derive own-price and cross-price elasticities for all products in the system.

Equation 7:

$$b_{jj} = \frac{\mathrm{sj}\ \sum_{m\neq j} \mathrm{Sm}\ \omega(\mathrm{j,m})}{\mathrm{s1} \sum_{m\neq 1} \mathrm{Sm}\ \omega(1,\mathrm{m})}\ \mathrm{b}_1$$

$$\mathrm{b_{ij}} = -\frac{\mathrm{s_i}\ \mathrm{s_j}}{\mathrm{s_1}} \frac{\omega\,(\mathrm{i,j})}{\sum_{\mathrm{k}=2}^{\mathrm{n}} \mathrm{s_k}\ \omega(\mathrm{k},1)} \mathrm{b}_{11},\ \mathrm{i} \neq \mathrm{j}$$

2.2 Data

Implementation of the model above requires data and observations on mail volumes, prices, revenues, and a general price index over time. The data on total sales are monthly figures coming from An Post's monthly sales MIS systems. The data are from 2001-2011 (December). Aggregate price indices, such as the CPI (monthly), the services CPI (monthly) and the CPI for postal services (monthly) were taken directly from the Central Statistics Office website.

[13] Coloma, G. (2006).

[14] This is not particular to the PCAIDS model. It is also true of the standard logit model. For this reason, often a nested logit or a conditional logit estimator will be chosen when there are observable differences between potential substitutes.

3 RESULTS

We present estimates from the LA-AIDS model first. These results can be used to generate the own-price elasticities needed for the PCAIDS model and we can also then compare and contrast the cross-price elasticity effects between the LA-AIDS and PCAIDS models.

Table 3.1: LA-AIDS Elasticity Estimates for Three-product system

	% Δ Price of		
% Δ Quantity of	Stamps	Metered	Bulk
Stamps	-0.22*	0.88	0.02
Metered	0.80	-0.39*	0.17
Bulk	0.07	0.18	-1.17

*Note: * indicates statistical significance*
Source: London Economics

In our first PCAIDS model, we assume that a similar three product market as before makes up the postal market. The aggregate price elasticity (Є) for the postal sector has been estimated previously and is quite inelastic (-0.17). From our previous aggregate time-series model estimations, we estimated that the price elasticity of stamped postage is -0.4 and this represents our product elasticity (ϵ_1) in the PCAIDS model. As we have full knowledge of the respective market shares within this market, we are able to derive the PCAIDS elasticities as per Equation 6. These are shown in Table 3.2 below.

The elasticities presented here are similar qualitatively, but statistically different from those reported previously. As the own price elasticity used for stamps is quite low and the industry elasticity is also low, there will be relatively little substitution between the various products in the market. From the results below, it appears that the bulk and metered postage are slightly closer substitutes when compared against stamped postage according to the PCAIDS model estimates.

Table 3.2 : PCAIDS Elasticity Estimates for Three-product system (calibrated on stamped postage)

	% Δ Price of		
% Δ Quantity of	Stamps	Metered	Bulk
Stamps	-0.39	0.10	0.12
Metered	0.10	-0.38	0.12
Bulk	0.12	0.12	-0.37

Source: London Economics

The results presented above were estimated by calibrating the demand system on the own-price elasticity of stamped postage. As a sensitivity check, we now perform the same estimation except this time we calibrate the model on the own-price elasticity for metered postage as estimated previously. The results of this are presented in Table 3.3.

The cross-price effects are lower in this model as the own-price elasticity of metered postage is estimated to be lower than the elasticity for stamped postage as per our LA-AIDS model. However, intuitively, the results appear somewhat consistent with the previous table as the cross-price impacts are higher for metered and bulk in comparison with stamped postage.

Table 3.3: PCAIDS Elasticity Estimates for Three-product system (calibrated on metered postage)

	% Δ Price of		
% Δ Quantity of	Stamps	Metered	Bulk
Stamps	-0.22	0.02	0.03
Metered	0.02	-0.22	0.03
Bulk	0.03	0.03	-0.22

Source: London Economics

4 SUMMARY AND CONCLUSIONS

We conclude that the PCAIDS method represents an interesting and potentially useful tool for the postal economist’s toolkit. It may be particularly useful when good estimates of overall industry demand elasticity exist but data limitations preclude estimation at a detailed product level. This may be particularly true in the postal industry, where for example data capture of items such as metering volumes, may not tabulate the exact volume for a customer by weight-step, format, and class.[15]

Comparing the LA-AIDS model with the PCAIDS model, shows that the PCAIDS model yielded less variation in the own price and cross price elasticities across the three product system. For example, the bulk postage own price elasticity was estimated at -1.17 via the LA-AIDS model, and -0.37 to -0.22 from the PCAIDS models.

Results from the exercise are interesting. It is difficult to compare the results with other papers because a full suite of demand elasticity parameters is usually not published. We compare our results with some papers and published reports that have published elasticity estimates using an alternative method.

As the industry price elasticity for postal services is inelastic, this will lead to relatively low cross-price elasticities for various postal products using a PCAIDS modelling framework. Using a nested PCAIDS model leads to higher cross-price elasticities for metered and bulk postage which are deemed, *a priori*, to be close substitutes.

[15] Many mail meters simply tally the total charge and pieces mailed, or it may be that MIS systems do not translate available data detail from each meter on weight, format and class.

While there appears to be some sensitivity to the choice of calibration own-price elasticity chosen, the result from our models show this sensitivity is not large. It is further possible to include a nesting structure within product categories to further limit the elasticities and substitution possibilities. This could prove particularly useful when considering difference across formats, for example. However, this would introduce further sensitivity of the model left to the practitioner, and it depends on whether a logical case can be made for the structure imposed.

The PCAIDS appears quite appropriate to estimate various cross-price elasticities in an observed product market, but would suggest additional research is needed to further define how it could be used with a more complicated set of postal product demand parameters.

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